

We start into the math for decay processes. Radioactive decay is a first order kinetic process and it follows the rate laws, integrated rate laws and half-life relationships as described in Chapter 49. If you need to, go back over Sections 49.1 - 49.3, especially the first order parts. Some of the lingo for nuclear kinetics is different, but the underlying math is the same.

### 67.1 Activity

The rate of radioactive decay is typically expressed in units which relate to individual decay events or some unit of those. Each decay event is called a disintegration (dis). Thus, a rate is commonly expressed as the number of disintegrations (decays) per time. One disintegration per second is a defined unit called a becquerel, Bq.

$$\text{Bq} = \text{one dis/s}$$

Historically, the curie (Ci) unit was very common for rate and it still is. Its numerical value was originally related to the decay rate of one gram of  $^{226}\text{Ra}$ , but the experimental value of that decay rate kept changing as methods of measurement got better. Nowadays a Curie is defined (exactly) in terms of Bq.

$$\text{Ci} = 3.7 \times 10^{10} \text{ dis/s} = 3.7 \times 10^{10} \text{ Bq}$$

Both Bq and Ci use per-second time units; you can actually use any time unit for rates, such as dis/h, but those units don't have a specific name. Like many units, Bq and Ci can take any of the standard numerical prefixes. For example, MBq is a megabecquerel and mCi is a millicurie.

OK, back to Chapter 49. The first order rate law was a simple proportionality.

$$\text{rate} = k [ ]$$

This rate is the speed of the reaction for a specific concentration of reactant. Now, for nuclear chemistry, this equation is modified as follows.

The word rate is replaced by activity,  $A$ . Activity is the speed of decay.

Rate constant,  $k$ , is not used; instead, the term "decay constant" is used and it gets  $\lambda$ .  $\lambda$  is the proportion of sample decaying per time.  $\lambda$  will have some unit of  $\text{time}^{-1}$ , just like a  $k$ .

Concentration is not used; instead, the sample as a whole is considered. A radioactive sample will contain some amount of a radionuclide, and that amount is designated  $N$ . Note that  $N$  is the amount of the radionuclide only; it could be present as an elemental form, as a compound, or even as a mixture of different compounds, but  $N$  is just the amount of just that nuclide undergoing decay. Fundamentally, the amount is expressed in terms of atoms of that nuclide, but this can be converted to moles, grams, or whatever, depending on the problem at hand.

With these new terms, the first order activity (rate) becomes

$$A = \lambda N$$

and this is still a simple proportionality. Thus, for a sample with twice the amount of radioactive nuclide, the activity is twice as much.

OK, now we have a kinetic  $A$  and  $N$  as used here, and we have  $A$  and  $N$  for mass number and neutron number. Same letters, different meaning. That might seem confusing. Don't blame me; I didn't do it.

In Chapter 49,  $k$  related directly to half-life.

$$t_{1/2} = \frac{\ln 2}{ak}$$

For nuclear decay,  $a$  is always one and  $k$  becomes  $\lambda$ .

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Keep in mind the qualitative relationships between activity (rate), decay constant and half-life as we continue.

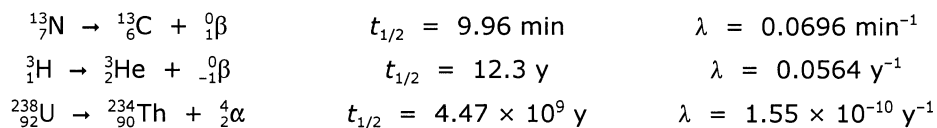
A faster rate of decay means a higher activity;

this gives a larger value for  $\lambda$ , meaning more decays per time; and,

$t_{1/2}$  is shorter, meaning less time is needed for half to decay.

A slower rate of decay means a lower activity;  
this gives a smaller value for  $\lambda$ , meaning fewer decays per time; and,  
 $t_{1/2}$  is longer, meaning more time is needed for half to decay.

Here are three examples of half-lives from the last Chapter, now with decay constants included.



These three nuclides decay at very different rates. For the same number of atoms,  ${}^{13}\text{N}$  decays the fastest and has the highest activity while  ${}^{238}\text{U}$  decays the slowest and has the lowest activity.

Let's start some calculations. I'll give you three pointers.

First pointer: Watch your time units. For example, if the problem wants dis/min and the  $\lambda$  or  $t_{1/2}$  is provided in  $\text{s}^{-1}$  or  $\text{s}$ , then you need to convert the time unit.

Second pointer: Watch your units for amount and activity. If you are given moles of  ${}^{238}\text{U}$  and you need Bq, then you have to convert mole to single atom scale using Avogadro. If you are given grams of  ${}^{238}\text{U}$  and you need Bq, then you have to convert grams to moles and moles to atoms; that means you'll need a molar mass and Avogadro.

Third pointer: You are dealing with one specific isotope of some element. As such, if you need a molar mass, you must use the molar mass for that specific isotope. In most cases, the molar mass values in the Periodic Table cannot be used because they are averaged over all natural isotopes according to abundance (as described in Section 2.3).

Let's start, using  ${}^{13}\text{N}$  from above.

**Example 1.** A sample contains 1.00 mmol (13.01 mg) of  ${}^{13}\text{N}$ .

- What is the activity of the sample in dis/min?
- What is the activity of the sample in Bq?
- What is the activity of the sample in Ci?

These all involve the activity equation, just going to different units in the end.

$$A = \lambda N$$

► Part A

You need an activity in dis/min. If you plug in  $\lambda$  and  $N$  as given, you have

$$A = \frac{0.0696}{\text{min}} \times 1.00 \text{ mmol}$$

but the units for activity in the answer would be mmol decaying per min. We need to take mmol down to atom level, and that needs Avogadro. To simplify, set 1.00 mmol to 0.00100 mol.

$$N = 0.00100 \text{ mol} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} = 6.022... \times 10^{20} \text{ atoms}$$

We're not rounding off yet. Put this into  $A$ .

$$A = \frac{0.0696}{\text{min}} \times 6.022... \times 10^{20} \text{ atoms}$$

The final units are the number of atoms per min, but keep in mind that  $\lambda$  is the decay constant. Thus, this calculation gives the number of atoms decaying per minute, which is the same as dis/min. Now, plug in, punch out, round off and you get  $4.19 \times 10^{19}$  dis/min.

► Part B

This is basically the same calculation, but Bq needs dis/s instead of dis/min. You need to convert the time unit in the decay constant.

$$\lambda = \frac{0.0696}{\text{min}} \times \frac{\text{min}}{60 \text{ s}}$$

Just plug this into A.

$$A = \frac{0.0696}{\text{min}} \times \frac{\text{min}}{60 \text{ s}} \times 6.022... \times 10^{20} \text{ atoms}$$

This gives  $6.99 \times 10^{17}$  dis/s, which is  $6.99 \times 10^{17}$  Bq.

► Part C

This follows Part B with one more conversion step.

$$A = \frac{0.0696}{\text{min}} \times \frac{\text{min}}{60 \text{ s}} \times 6.022... \times 10^{20} \text{ atoms} \times \frac{\text{Ci}}{3.7 \times 10^{10} \text{ Bq}} = 1.89 \times 10^7 \text{ Ci}$$

The 3.7 in the Curie conversion is exact, so the rounding is to three sigfigs as done in Parts A and B.

The  $^{13}\text{N}$  here was the fastest decayer of the three listed above. Note the huge number of decays per second (Part B) for only a mmol of material. For comparison, let's check out the laggard from the above list,  $^{238}\text{U}$ . We'll just do Bq for the comparison and then I'll throw in a twist.

**Example 2.** Consider the  $\alpha$  decay of  $^{238}\text{U}$ .

A. What is the activity (in Bq) of 1.00 mmol of  $^{238}\text{U}$ ?

B. What is the activity (in Bq) of 1.00 mg  $^{238}\text{U}$ ? The molar mass of  $^{238}\text{U}$  is 238.1 g.

Both Parts will start the same way.

$$A = \lambda N$$

► Part A.

Plug in  $\lambda$  and  $N$ .

$$A = \frac{1.55 \times 10^{-10}}{\text{y}} \times 1.00 \text{ mmol}$$

How do the units shape up? Right now, they are at mmol (decaying) per year and we need dis/s. As in the prior Example 1, mmol must be taken to atom level.

$$N = 0.00100 \text{ mol} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} = 6.022... \times 10^{20} \text{ atoms}$$

We must also again convert the time unit in  $\lambda$ , but now from  $\text{year}^{-1}$  to  $\text{second}^{-1}$ .

$$\lambda = \frac{1.55 \times 10^{-10}}{\text{y}} \times \frac{\text{y}}{365.24 \text{ d}} \times \frac{\text{d}}{24 \text{ h}} \times \frac{\text{h}}{3600 \text{ s}} = 4.9117... \times 10^{-18} \text{ s}^{-1}$$

Note the 365.24 days in a year. For calculations over many many years, the value 365.24 is used. This takes into account leap years and it takes into account that 3-out-of-4 century years are officially not leap years. (A century year is, e.g., 2000, 1900, 1800, 1700, etc. Every fourth century year, such as 2000, is a leap year, but the others are not.) Yet other factors also enter into 365.24. The value 365.24 is not exact and many more decimal places are known, but this is enough for our purposes.

Now combine  $\lambda$  and  $N$ .

$$A = \lambda N = 4.9117... \times 10^{-18} \text{ s}^{-1} \times 6.022... \times 10^{20} \text{ atoms} = 2,960 \text{ dis/s} = 2,960 \text{ Bq}$$

This is a slow rate of decay in comparison to the decay for one mmol of  $^{13}\text{N}$  in Example 1, Part B.

► Part B

Now, start with a mass for  $N$ .

$$A = \frac{1.55 \times 10^{-10}}{\text{y}} \times 1.00 \text{ mg}$$

How are the units? They are currently mg (decaying) per year. We'll need to convert both  $N$  and  $\lambda$  again. The time conversion for  $\lambda$  is the same as in Part A, so we again use  $\lambda = 4.9117... \times 10^{-18} \text{ s}^{-1}$ . For  $N$ , however, we now start from grams. That will go through the standard old routine of g to mol via molar mass, then mol to atoms via Avogadro. Note the molar mass for  $^{238}\text{U}$  is given.

$$N = 0.00100 \text{ g} \times \frac{\text{mol}}{238.1 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} = 2.5291... \times 10^{18} \text{ atoms}$$

Combine  $\lambda$  and  $N$ .

$$A = \lambda N = 4.9117... \times 10^{-18} \text{ s}^{-1} \times 2.5291... \times 10^{18} \text{ atoms} = 12.4 \text{ dis/s} = 12.4 \text{ Bq}$$

Finis.

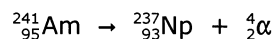
Next. We'll do a radioactive nuclide which people buy for their homes.

**Example 3.**  $^{241}\text{Am}$  undergoes alpha decay with  $t_{1/2} = 433 \text{ y}$ . What is the activity (in  $\mu\text{Ci}$ ) of  $0.200 \mu\text{g}$  of  $^{241}\text{Am}$ ? The molar mass of  $^{241}\text{Am}$  is  $241.1 \text{ g}$ .

People buy  $^{241}\text{Am}$  for their homes? Yes, this was first mentioned in the Group tours of Section 2.5.

“...americium (Am) is used in the common, ionization smoke detectors. Yes, you may have a nuclear device right in your own home. The americium does its nuke thing in the sensor chamber to detect the smoke. It's  $^{241}\text{Am}$ . Next time you change the batteries (at least once or even twice a year AND DON'T FORGET!), you may notice the nuclear warning labels inside but don't go messing with it.”

We can now describe how these types of smoke detectors operate. The decay equation was given in Chapter 66.



Inside the smoke detector is a chamber which contains  $^{241}\text{Am}$ , emitting  $\alpha$  particles into the air in the chamber. The chamber has slots or holes to allow room air to diffuse in and out. The  $\alpha$  particles cause the air inside the chamber to ionize. The electronics set up positive and negative electrodes within the chamber, and that sets up for ion flow. Ion flow equals current, even in a gas phase, and the electronics monitor that current. When smoke particles get into the chamber, they intercept many of the ions and decrease the current; the electronics sense that, and sound the alarm. Other things can also change the current and sound the alarm, such as fine dust or even a fog from a shower. Americium does not exist naturally on Earth; all  $^{241}\text{Am}$  used in smoke detectors is made by a series of nuclear reactions and decays (Section 69.9). When manufactured, a smoke detector typically contains  $0.9 - 1.0 \mu\text{Ci}$ . These things save a lot of lives.

For the problem at hand, you seek the activity for the given amount

$$A = \lambda N$$

but now  $\lambda$  is not given. You'll need to derive that from  $t_{1/2}$ , which is given.

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Re-arrange the latter

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

and substitute into the former.

$$A = \frac{\ln 2}{t_{1/2}} N$$

Plug in the given values.

$$A = \frac{\ln 2}{433 \text{ y}} \times 0.200 \mu\text{g}$$

What conversions are needed? As in Example 2, Part B, we need to take mass down to atom level.

$$2.00 \times 10^{-7} \text{ g} \times \frac{\text{mol}}{241.1 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} = 4.9954... \times 10^{14} \text{ atoms}$$

We also need a bit of work with the time unit. For  $\mu\text{Ci}$ , you'll need the time unit to be s, but  $t_{1/2}$  has the time unit of y. Convert.

$$t_{1/2} = 433 \text{ y} \times \frac{365.24 \text{ d}}{\text{y}} \times \frac{24 \text{ h}}{\text{d}} \times \frac{3600 \text{ s}}{\text{h}} = 1.3664... \times 10^{10} \text{ s}$$

Now plug everything in.

$$A = \frac{\ln 2}{t_{1/2}} \times N = \frac{\ln 2}{1.3664... \times 10^{10} \text{ s}} \times 4.9954... \times 10^{14} \text{ atoms}$$

The units of activity are now atoms (decaying) per s, or dis/s, which is Bq. You still need another conversion to get to Ci, and then don't forget the question asks for  $\mu\text{Ci}$ . Once you've done all of these things, you should arrive at 0.685  $\mu\text{Ci}$ .

These Examples illustrate the basic calculations of activity for some amount of some radioactive nuclide. Variations are possible, such as calculating an amount of substance based on a given activity. These are plug-ins to the proportionality equation, although the unit conversions can be tedious.

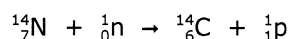
## 67.2 Human activity

Your smoke detector is not the only item in your house which is undergoing radioactive decay. You are, too. Naturally.

Except for water, many of the most common compounds in biological systems contain carbon. Sugars, proteins, fats, DNA, etc., all contain carbon (and other elements also). As noted in the last Chapter, there's a trace amount of radioactive  $^{14}\text{C}$  naturally on Earth.



With a half-life of only 5,700 y, the Earth's supply of  $^{14}\text{C}$  would not last on its own through the billions of years of Earth's history, but additional  $^{14}\text{C}$  is constantly being made by a nuclear reaction in the upper atmosphere. What's that? A nuclear reaction? In the atmosphere? Actually, there are numerous, natural, nuclear reactions going on constantly in the atmosphere, as a result of bombardment by cosmic rays. Cosmic rays are very high energy protons and  $\alpha$  particles, along with lesser amounts of other nuclei and subatomic particles. These originate from a multitude of sources across the cosmos, striking Earth from every direction. Their energies can be extreme. Upon collision with some atom or molecule in the upper atmosphere, a large sequence of particles can be produced as the energy of the original cosmic ray is dissipated. Neutrons are part of this picture, and such neutrons can strike an atom of  $^{14}\text{N}$  (which is the most abundant nuclide in the atmosphere as part of  $\text{N}_2$ ), forming an atom of  $^{14}\text{C}$  and spitting out a proton in the process.



This nuclear reaction constantly replenishes Earth's decaying supply of  $^{14}\text{C}$ . Thus, some  $^{14}\text{C}$  is coming and some  $^{14}\text{C}$  is going at all times and a balance results, although there are fluctuations in the balance over the centuries and millennia due to changes in the amount of cosmic rays. So far, all of this is natural. In addition, humans have interfered with the natural balance of  $^{14}\text{C}$  in their own way. Since  $\sim 1950$ , there have been various releases of  $^{14}\text{C}$  into the atmosphere from human-derived nuclear processes (e.g., bomb tests and other activities). On the other hand, the burning of fossil fuels has released an overwhelming amount of  $\text{CO}_2$  into the atmosphere and this  $\text{CO}_2$  has no  $^{14}\text{C}$  in it. The fossil fuels themselves are millions of years old, and their original  $^{14}\text{C}$  content has long ago decayed into oblivion. Regardless, whether with or without human interference, there is always some  $^{14}\text{C}$  present on Earth, although it is in extremely minute amounts. Ordinary natural carbon is  $^{12}\text{C}$  (98.93%) and  $^{13}\text{C}$  (1.07%);  $^{14}\text{C}$  is only about  $10^{-10}\%$  of all carbon atoms.

$^{14}\text{C}$  in the atmosphere from cosmic ray bombardment will eventually react chemically to form  $^{14}\text{CO}_2$ . That  $^{14}\text{CO}_2$  just mixes in with all the ordinary  $\text{CO}_2$  in the air, eventually making its way to Earth's surface.  $^{14}\text{CO}_2$  does the same chemical reactions as ordinary  $\text{CO}_2$ . That includes being absorbed by plants which convert  $\text{CO}_2$  to sugars. Many plants are then eaten by animals; plants and animals are eaten by humans. All along the way, the atoms of  $^{14}\text{C}$  are being passed on, incorporated into a variety of biological molecules such as fats, proteins, DNA, other sugars, etc. Some is exhaled again as  $\text{CO}_2$  and some is excreted, but your body retains some  $^{14}\text{C}$  throughout your system, still  $\sim 10^{-10}\%$  of your total carbon atoms. In this

way, you, I, and all life forms on Earth, including that pesky fly buzzing around the room, are naturally decaying.

But how much so?

Humans are ~18% carbon by mass. For a human of 68.0 kg (150. lb) body weight, this corresponds to 12 kg total C. We'll do a ballpark calculation using an estimate of  $1.2 \times 10^{-12}$  g  $^{14}\text{C}$  for every g of total C in a living sample. This means that a 150. lb human contains  $1.4 \times 10^{-8}$  g of  $^{14}\text{C}$ . Not much in grams, eh? What's that person's activity level?

.....  
**Example 4.** What is the activity (in dis/min) of  $1.4 \times 10^{-8}$  g  $^{14}\text{C}$ ? The molar mass for  $^{14}\text{C}$  is 14.00 g.  
 .....

Set it up.

$$A = \lambda N$$

We have  $t_{1/2}$  from above but not  $\lambda$ . Convert.

$$A = \frac{\ln 2}{t_{1/2}} N$$

Set up the unit conversions.

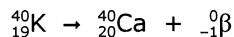
$$\frac{\ln 2}{t_{1/2}} : \quad \frac{\ln 2}{5,700 \text{ y}} \times \frac{\text{y}}{365.24 \text{ d}} \times \frac{\text{d}}{24 \text{ h}} \times \frac{\text{h}}{60 \text{ min}} = 2.3121... \times 10^{-10} \text{ min}^{-1}$$

$$N: \quad 1.4 \times 10^{-8} \text{ g } ^{14}\text{C} \times \frac{\text{mol } ^{14}\text{C}}{14.00 \text{ g } ^{14}\text{C}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol } ^{14}\text{C}} = 6.022... \times 10^{14} \text{ atoms}$$

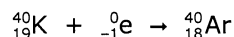
Multiplying these two together gives 140,000 atoms (decaying) per min, or 140,000 dis/min. If you weigh 150 lb, you are disintegrating 140,000 times each minute just due to your  $^{14}\text{C}$  content.

Now, to add further to your decay, consider potassium.

Natural potassium includes radioactive  $^{40}\text{K}$ , which decays by two routes: 89.28% via  $\beta^-$  emission



and 10.72% by EC.



Either route is very slow: taking both into account, the overall  $t_{1/2} = 1.25 \times 10^9$  y. The long half-life allows for some original  $^{40}\text{K}$  to still exist from when Earth was formed, although most has since decayed. The current natural abundance of  $^{40}\text{K}$  is only 0.0117% out of all K. The products of the above decays,  $^{40}\text{Ca}$  and  $^{40}\text{Ar}$ , are not radioactive. The  $^{40}\text{Ca}$  which is formed will end up chemically as  $\text{Ca}^{2+}$  and stay dissolved in solution or within some solid such as rock or even bone. The  $^{40}\text{Ar}$  which is formed ends up as neutral noble gas, not reacting chemically with anything else. Eventually, it makes its way to the atmosphere, where it does nothing except accumulate. Argon is 0.9% of dry air. The overwhelming majority (99.60%) of all Ar in air is  $^{40}\text{Ar}$ , due primarily to  $^{40}\text{K}$  decay over several billion years.

Back to you. Overall, you are ~0.20% K by mass. Most potassium is inside your cells where  $\text{K}^+$  is a major electrolyte. A 150. lb human has a total of 140 g K, of which 16 mg is  $^{40}\text{K}$ .

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**Example 5.** What is the activity of 16 mg  $^{40}\text{K}$  in dis/min? The molar mass of  $^{40}\text{K}$  is 39.96 g.  
 .....

Your turn.

This comes to 250,000 dis/min, which is a higher activity than from  $^{14}\text{C}$ .

You have other radioactive things in you also, but their contributions are much smaller than from these two. If you combine just the  $^{14}\text{C}$  and  $^{40}\text{K}$  within your body, you are disintegrating on the order of 390,000 times per minute. Mostly  $\beta^-$ , some EC. All because you live on a radioactive planet. Naturally.

### 67.3 Integration of disintegration

So far, all calculations have been based on the simple proportionality

$$A = \lambda N$$

in direct parallel to the first order rate law of Chapter 49.

$$\text{rate} = k [ ]$$

Now we bring in the integrated rate equation. In Chapter 49, the first-order equation was

$$\ln \frac{[ ]_0}{[ ]_t} = akt$$

in which  $[ ]_0$  and  $[ ]_t$  were the concentrations at initial time zero and some later time,  $t$ . For present purposes, we again use amounts instead of concentrations,  $a$  is still one, and we use  $\lambda$  instead of  $k$ .

$$\ln \frac{N_0}{N_t} = \lambda t$$

Since  $A = \lambda N$ , we can write  $A_0 = \lambda N_0$  and  $A_t = \lambda N_t$ ; the ratio of amounts then equals the ratio of activities

$$\frac{A_0}{A_t} = \frac{\lambda N_0}{\lambda N_t} = \frac{N_0}{N_t}$$

and the integrated rate equation can be written in either form.

$$\ln \frac{N_0}{N_t} = \ln \frac{A_0}{A_t} = \lambda t$$

This allows you to use the ratio of the amounts or the ratio of the activities, depending on the problem at hand. Furthermore, you can use any unit for the  $N$ 's and any unit for the  $A$ 's, as long as it's the same unit upstairs and downstairs. You can get away with using any unit because the units drop out in the ratio. For a given decay,  $\lambda$  is a constant; otherwise, this relationship involves three variables: initial condition (amount or activity), final condition (amount or activity), and time. If you know two, you can calculate the third.

Let's proceed with some of these.

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**Example 6.** Start with the sample in Example 1: 1.00 mmol  $^{13}\text{N}$  with  $A = 6.99 \times 10^{17}$  Bq.

- A. How much  $^{13}\text{N}$  (in mmol) remains after 1.50 h?  
 B. What is the activity (in Bq) of the  $^{13}\text{N}$  after that 1.50 h?
- .....

Both Parts are based on the integrated rate equation.

$$\ln \frac{N_0}{N_t} = \ln \frac{A_0}{A_t} = \lambda t$$

► Part A

We are given a starting amount,  $N_0$ , and we seek the amount remaining,  $N_t$ , after 1.50 h.

$$\ln \frac{N_0}{N_t} = \lambda t$$

Re-arrange for  $N_t$ .

$$\frac{N_0}{N_t} = e^{\lambda t}$$

$$N_t = N_0 e^{-\lambda t}$$

How do the units shape up? Both  $N$ 's are in mmol, so they can stay that way. The  $\lambda$  has  $\text{min}^{-1}$  and  $t$  has h; they need to have the same unit, and it could be either min or h. Here's the  $\lambda t$  part with the time conversion.

$$\lambda t = \frac{0.0696}{\text{min}} \times \frac{60 \text{ min}}{\text{h}} \times 1.50 \text{ h} = 6.264\dots$$

Plug that into the whole thing.

$$N_t = N_0 e^{-\lambda t} = 1.00 \text{ mmol} \times e^{-6.264\dots}$$

Punch out and round off; you get 0.00190 mmol of  $^{13}\text{N}$  remaining after 1.50 h. That's a mere 0.190% remaining after only 1.50 h, which means that 99.810% has decayed. This is another illustration of how fast  $^{13}\text{N}$  decays.

► Part B

You now have two choices.

First choice: Part A showed that 0.00190 mmol  $^{13}\text{N}$  is present after the 1.50 h; you can use that in the proportionality equation.

$$A = \lambda N$$

Second choice: The starting activity was  $6.99 \times 10^{17}$  Bq. You can now use the integrated equation again, setting it up for activity.

$$\ln \frac{A_0}{A_t} = \lambda t$$

Here is the setup for the first choice

$$A = \frac{0.0696}{\text{min}} \times 1.90 \times 10^{-6} \text{ mol} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} \times \frac{\text{min}}{60 \text{ s}}$$

and here it is, after re-arranging, for the second choice.

$$A_t = A_0 e^{-\lambda t} = 6.99 \times 10^{17} \text{ Bq} \times e^{-6.264\dots}$$

Either way, you get  $1.33 \times 10^{15}$  Bq for the activity after 1.50 h, which is again only 0.190% of the starting activity.

Compare this to  $^{238}\text{U}$ . We'll just do the amount remaining for this one.

.....  
**Example 7.** Start with the sample in Example 2, Part A: 1.00 mmol  $^{238}\text{U}$ . How much  $^{238}\text{U}$  (in mmol) remains after 1.50 h?  
 .....

You need

$$\ln \frac{N_0}{N_t} = \lambda t$$

which again re-arranges to

$$N_t = N_0 e^{-\lambda t}$$

and again you need to convert the time units. Here's the  $\lambda t$  part with the time conversion.

$$\lambda t = \frac{1.55 \times 10^{-10}}{\text{y}} \times \frac{\text{y}}{365.24 \text{ d}} \times \frac{\text{d}}{24 \text{ h}} \times 1.50 \text{ h} = 2.6523\dots \times 10^{-14}$$

Plug in.

$$N_t = N_0 e^{-\lambda t} = 1.00 \text{ mmol} \times e^{-2.6523\dots \times 10^{-14}}$$

Punch out, round off; you get 1.00 mmol remaining, which is the same as the initial amount. This means that no significant amount of the  $^{238}\text{U}$  decayed in the 1.50 h; it's just too slow. (The change in mmol would be way out in the fourteenth decimal place, and would not be significant.)



One final Example. They say to replace your smoke detectors every ten years, and preferably sooner. Is this because the Am is running out? Ten years is far short of the half-life, 433 y, so far less than half of the amount will decay in 10 y, but you can calculate just how much does remain. This one's yours.

**Example 8.** For a smoke detector which starts with 1.00  $\mu\text{Ci}$ , how many  $\mu\text{Ci}$  remain after 10.0 y?

Start with

$$\ln \frac{A_0}{A_t} = \lambda t$$

and solve for  $A_t$ . You don't have  $\lambda$ , so bring in the  $t_{1/2}$  step. The units here are straightforward and no unit conversions are needed.

Go.

You will find that  $A_t$  has not changed much (only 1.6%) from  $A_0$ . So, no, your Am is not running out much in 10 years. The timeframe for replacement is based on typical electronics. How many electronic devices do you bet your life on after 10 years?

Let me repeat two sentences from earlier above regarding the integrated rate equation.

For a given decay,  $\lambda$  is a constant; otherwise, this relationship involves three variables: initial condition (amount or activity), final condition (amount or activity), and time. If you know two, you can calculate the third.

So far, we've solved for some final amount or activity after some time into the future. Solving for an initial amount or activity would parallel that approach. But you can also solve for time, and that can get you a date.

**Example 9.** A sample of tritium which was prepared decades ago was known to have an original activity of 42.3 mCi when first produced. You measure the present activity and find that it is 8.11 mCi. How long ago (in y) was the sample produced?

You need

$$\ln \frac{A_0}{A_t} = \lambda t$$

for which you are given  $A_0$  and  $A_t$ ;  $\lambda$  is in the listing earlier,  $0.0564 \text{ y}^{-1}$ . You're looking for the time which has elapsed since the sample was produced with the given activity of  $A_0$ . Re-arrange the equation to solve for  $t$ .

$$t = \frac{1}{\lambda} \times \ln \frac{A_0}{A_t} = \frac{1}{0.0564 \text{ y}^{-1}} \times \ln \frac{42.3 \text{ mCi}}{8.11 \text{ mCi}} = 29.3 \text{ y}$$

This says that the sample was produced 29.3 y ago.

This is a simple illustration of a calculation going back in time, based on a radioactive decay. This is the basis for the field of radiochemical dating, which determines ages for a huge variety of objects, inanimate and (formerly) animate. The methods have been applied to rocks (Earth and Moon), meteorites, waterways, ice cores, and samples of biological origin. The method can determine ages from dozens of years to billions of years in age, depending on the presence in the object of a suitable radioactive nuclide, even if it's only at a small level.

Here're the basics of how this works. The initial  $t_0$  is the time the object was formed; at that point there was some initial amount,  $N_0$ , and activity,  $A_0$ , of some radionuclide present. That amount and activity has decreased due to decay, over whatever number of years the object has existed, up to the

present time. That number of years is  $t$ . At the present time, there is a remaining amount or activity of that nuclide, and that is  $N_t$  or  $A_t$ . This part can be measured directly. There must be some way of knowing the original  $N_0$  or  $A_0$ , and this is typically the tricky part although there are ways of assessing that. For example, in many rock samples, the parent and the daughter are trapped together, so their sum (in atoms or mol) is the original amount. Putting all of this together,

if you know the original  $N_0$  or  $A_0$  of the object,  
and, if you measure the present value of  $N_t$  or  $A_t$ ,

then you can determine the amount of time,  $t$ , which has elapsed using the integrated rate equation. That amount of time is the age of the object.

A considerable number of radioactive nuclides can be used for dating, covering a considerable range of ages. Here are some examples.



You can also use various members of a radioactive decay series such as the  $^{238}\text{U}$  series; in fact, you can even use the entire  $^{238}\text{U}$  series all at once, by measuring the amounts of  $^{238}\text{U}$  and  $^{206}\text{Pb}$  in a sample. Given the many parent/daughter combinations which can be used for radioactive dating, it is possible to measure from more recent times (decades) up to billions of years ago.

One of the most important nuclides for dating biological specimens is  $^{14}\text{C}$ , and this has been the most important for dating early human history. The use of  $^{14}\text{C}$  is specifically called radiocarbon dating. As described in the last Section, there is an incredibly small amount of  $^{14}\text{C}$  present on Earth and that is constantly incorporated into various compounds within a living organism. As long as the organism is living, it takes in trace amounts of  $^{14}\text{C}$  and it exhales or excretes trace amounts of  $^{14}\text{C}$  as part of its normal carbon balance. Once that organism dies, then intake ceases; at the time of death, the body of the organism contains some initial ratio of  $^{14}\text{C}$  relative to its total carbon content. That time of death is time zero for dating purposes. As time passes on, the  $^{14}\text{C}$  continues to decay, so the ratio of  $^{14}\text{C}$  to total C decreases. If the remains of that organism are examined hundreds of years later, or thousands of years later, the amount of  $^{14}\text{C}$  can be measured relative to the total C which is still present. That leads to the amount of time which has passed since the death of that organism. Even if the organism was killed and used for something, such as a plant made into cloth or a bone made into a tool, the cloth or tool still carries the carbon information.

$^{14}\text{C}$  dating has been extremely valuable for chronicling many aspects of human history, including the migration of humans around the globe, cave paintings, writings from one thousand or several thousand years ago, and even documents or artifacts of only hundreds of years of age.  $^{14}\text{C}$  dating has also been used for nonhuman events. Unfortunately, the method is more complicated than what can be covered here. The reason for that is the fact that variations in the natural  $^{14}\text{C}$  balance over the many years do need to be taken into account. The integrated rate equation is not adequate by itself, and calibration curves must be used. Using sophisticated calibrations from a wide variety of sources, radiocarbon dating is being used for determining ages as far back as ~55,000 years ago. While there is more uncertainty in the older times, improvements to the calibrations continue to be made.

OK, so much for death and decay. It's time to move on.

### Problems

- $^{44\text{m}}\text{Sc}$  decays by  $\gamma$  emission, with  $\lambda = 0.0118 \text{ h}^{-1}$ . Consider a sample which contains  $1.19 \mu\text{mol}$  of  $^{44\text{m}}\text{Sc}$ .
  - What is the activity of the sample in Bq?
  - What is the activity of the sample in Ci?
- $^{32}\text{P}$  has been used in biological and medical applications for many years. This nuclide undergoes  $\beta^-$  decay with  $t_{1/2} = 14.3 \text{ d}$ . What is the activity (in dis/min) of a sample containing  $4.87 \mu\text{g}$   $^{32}\text{P}$ ? The molar mass is  $31.97 \text{ g}$ .

3. The activity of a typical banana due to  $^{40}\text{K}$  is 13.1 Bq. What mass of  $^{40}\text{K}$  (in g) is present? The molar mass of  $^{40}\text{K}$  is 39.96 g and  $t_{1/2} = 1.25 \times 10^9$  y.
4.  $^{67}\text{Ga}$  decays by electron capture, with  $t_{1/2} = 3.26$  d.
  - a. What is the activity (in Ci) of a sample containing  $2.86 \mu\text{mol } ^{67}\text{Ga}$ ?
  - b. What will be the activity (in Ci) of that sample after 21.0 d?
5.  $^{236}\text{Pu}$  decays by  $\alpha$  emission, with  $\lambda = 0.243 \text{ y}^{-1}$ . The molar mass of  $^{236}\text{Pu}$  is 236.0 g. A sample containing  $42.6 \mu\text{g } ^{236}\text{Pu}$  is prepared. How much  $^{236}\text{Pu}$  (in  $\mu\text{g}$ ) will remain after 14.0 y?
6. The PET radionuclide  $^{18}\text{F}$  decays with  $t_{1/2} = 110$ . min. If a sample is prepared with an activity of 100. MBq, what is the activity (in MBq) after 8.00 h?
7.  $^{87}\text{Rb}$  decays by  $\beta^-$  emission with  $t_{1/2} = 4.97 \times 10^{10}$  y, and it is useful in dating rocks and meteorites. Consider a rock sample which contains  $2.17 \mu\text{mol } ^{87}\text{Rb}$  and  $0.130 \mu\text{mol } ^{87}\text{Sr}$ , and assume that all of the  $^{87}\text{Sr}$  came from decay of the  $^{87}\text{Rb}$ . What is the age (in y) of the sample?