

Chapter 69 NUCLEAR CHEMISTRY, Part 4

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We begin with a new aspect related to mass-energy.

In Chapter 66, we distinguished stable versus unstable nuclides simply by whether a nuclide has been observed to decay or not. There is actually a range for "stable" as well as for "unstable", and we can use the mass-energy relationship to examine those ranges.

69.1 Mass defect and binding energy

Section 2.3:

“ One u is an incredibly small mass, 1.6605×10^{-24} g, and this is close to the mass of a single proton or neutron: 1.0073 u for a proton and 1.0087 u for a neutron. (In comparison, an electron mass is only 0.00055 u.) Let's look at some examples of actual masses for atoms.

One atom of ^{19}F has an actual mass of 18.998403 u, which is close to its mass number, 19.

One atom of ^{35}Cl has an actual mass of 34.968853 u, which is close to its mass number, 35.

One atom of ^{37}Cl has an actual mass of 36.965903 u, which is close to its mass number, 37.

One atom of ^{197}Au has an actual mass of 196.966570 u, which is close to its mass number, 197.

Since electrons contribute very little mass relative to protons and neutrons, the true mass is very close to the mass number. In most cases, the true mass is a bit less than the mass number due to another factor, called mass defect. We'll discuss mass defect much later, in Chapter 69. ”

Later is now.

As done in Chapter 68, we will work with more sigfigs for masses in this Chapter, and that will now include masses for protons, neutrons and electrons. Here are their masses to six decimal places; more decimals are known, but this is enough for our purposes.

$$p^+ \quad 1.007276 \text{ u} \quad n^0 \quad 1.008665 \text{ u} \quad e^- \quad 0.000549 \text{ u}$$

For every atom with two or more nucleons, if you add up the masses for all of the protons, neutrons and electrons, then you will get a sum which is greater than the actual mass of that atom. The difference between the summed masses of the separate particles and the actual mass of the atom is the mass defect.

$$\text{mass defect} = \text{sum of separate particle masses} - \text{actual mass}$$

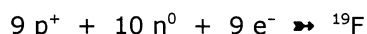
Because the sum is always greater than the actual mass, the mass defect is always a positive number. For one example, consider ^{19}F : one atom of ^{19}F has 9 p^+ , 10 n^0 and 9 e^- . Add up the masses for those numbers of those individual particles.

$$(9 \times 1.007276 \text{ u}) + (10 \times 1.008665 \text{ u}) + (9 \times 0.000549 \text{ u}) = \\ 9.065484 \text{ u} + 10.086650 \text{ u} + 0.004941 \text{ u} = 19.157075 \text{ u}$$

(For sigfig purposes, I'm going to invoke the +/- rule again, as was done for similar calculations for molar masses way back in Chapter 5. Since our input masses are to six decimals, then calculations of this type will give answers to six decimals.) As noted and quoted above, the actual mass of one atom of ^{19}F is 18.998403 u. The actual mass of one atom of ^{19}F is less than the sum of the particle masses. The difference is the mass defect.

$$\text{mass defect} = 19.157075 \text{ u} - 18.998403 \text{ u} = 0.158672 \text{ u}$$

Here is the meaning and the significance. Consider one atom to form from its individual, constituent particles.

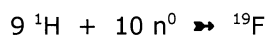


In the process, a huge amount of energy would be released and this carries a considerable amount of mass with it. As a result, the product atom has considerably less mass than the total of the starting masses. The change in mass between initial state (individual, separate particles) and the final state (one atom) is the mass defect.

Typically, the calculation is simplified by using the mass of one atom of ^1H to count for one p^+ plus one e^- . Thus, the numbers to use will involve only the masses for ^1H and n^0 .



The process can then be represented as



and the calculation is now

$$(9 \times 1.007825 \text{ u}) + (10 \times 1.008665 \text{ u}) = 9.070425 \text{ u} + 10.086650 \text{ u} = 19.157075 \text{ u}$$

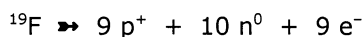
which gives the same sum as above. This will lead to the same value for the mass defect, 0.158672 u.

The energy of this hypothetical process of forming an atom from its particle parts can be calculated from the mass defect, using the 931.5 conversion factor as a shortcut. For one atom of ${}^{19}\text{F}$,

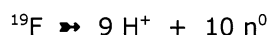
$$E = 0.158672 \text{ u} \times \frac{931.5 \text{ MeV}}{\text{u}} = 147.8 \text{ MeV}$$

and that is an absolutely huge amount of energy. Again, this is a hypothetical process; this is NOT the way the atoms are actually formed, but these calculations have a lot of conceptual utility. For a given number of nucleons as given by the mass number, A , the energy of the mass defect is a measure of how strongly the nucleus is held together. In fact, this energy is called the binding energy. For a more strongly bound nucleus, there will be a greater mass defect, giving a greater binding energy.

You can also consider the process in reverse; for example, how much energy would it take to pull one atom of ${}^{19}\text{F}$ completely apart into its constituent particles?



OR



That would be 147.8 MeV.

To illustrate how big these energies are, let's put this again into the perspective of previous coverage of chemical energies, on a per-mole basis. We'll do the $E = mc^2$ calculation; for this purpose, the 0.158672 u for one atom means 0.158672 g for one mol of those atoms.

$$E = mc^2 = 1.58672 \times 10^{-4} \text{ kg} \times (3.00 \times 10^8 \text{ m/s})^2 = 1.43 \times 10^{13} \text{ J} = 14.3 \text{ TJ}$$

TJ? That's terajoules, 10^{12} J, a trillion joules. Binding energies are even more outlandish than decay energies. The notion of nuclear binding energy has strong parallel to chemical bond energies. Compare the following conceptual processes.

Chemical process:

Individual atoms come together to form a molecule. Energy is released, corresponding to the bond energies. That amount of energy would be needed to pull the molecule apart into its separate atoms.

Nuclear process:

Individual protons, neutrons and electrons join to form an atom. Energy is released, corresponding to the binding energy. That amount of energy would be needed to pull the atom apart into its separate protons, neutrons and electrons.

Although conceptual parallel can be drawn, the energies are horrendously different between the two types of processes.

Let's now compare the binding energy of ${}^{19}\text{F}$ to that of ${}^{19}\text{Ne}$, which has the same total number of nucleons. Find the mass defect and the binding energy of ${}^{19}\text{Ne}$, given that the actual mass of ${}^{19}\text{Ne}$ is 19.001881 u.

In order to do this, you need the total mass of ten ${}^1\text{H}$ and nine n^0 .

$$(10 \times 1.007825 \text{ u}) + (9 \times 1.008665 \text{ u}) = 19.156235 \text{ u}$$

This leads to the mass defect

$$19.156235 \text{ u} - 19.001881 \text{ u} = 0.154354 \text{ u}$$

which leads to the binding energy.

$$E = 0.154354 \text{ u} \times \frac{931.5 \text{ MeV}}{\text{u}} = 143.8 \text{ MeV}$$

Notice that the mass defect and the binding energy for ^{19}Ne are less than those values for ^{19}F . We would judge ^{19}F to be the more strongly bound nuclide and the more stable nuclide. In fact, ^{19}Ne is radioactive (β^+ decay; $t_{1/2} = 17.3 \text{ s}$) and not stable at all.

Every nuclide except one has a mass defect and a binding energy. The lone one has a single proton by itself with nothing else in the nucleus: it's just ^1H . For every other nuclide, their actual mass is less than the sum of the masses of the separate particles of which they are composed. Let's take a look at two more nuclides.

Example 1. Calculate the mass defect (in u) and the binding energy (in MeV) for ^{53}Cr (actual mass, 52.940646 u) and for ^{75}As (actual mass, 74.921595 u).

We envision one atom of ^{53}Cr to be built from 24 atoms of ^1H and 29 neutrons. The total mass for these separate particles is

$$(24 \times 1.007825 \text{ u}) + (29 \times 1.008665 \text{ u}) = 53.439085 \text{ u}$$

and the mass defect is

$$53.439085 \text{ u} - 52.940646 \text{ u} = 0.498439 \text{ u}$$

for which we can calculate the binding energy.

$$E = 0.498439 \text{ u} \times \frac{931.5 \text{ MeV}}{\text{u}} = 464.3 \text{ MeV}$$

You can do ^{75}As . How many ^1H and n^0 masses?

$$(\text{_____} \times 1.007825 \text{ u}) + (\text{_____} \times 1.008665 \text{ u}) = \text{_____} \text{ u}$$

What is the mass defect?

$$\text{_____} \text{ u} - 74.921595 \text{ u} = \text{_____} \text{ u}$$

What is the binding energy?

$$E = \text{_____} \text{ u} \times \frac{931.5 \text{ MeV}}{\text{u}} = \text{_____} \text{ MeV}$$

That comes to 652.6 MeV. These binding energies are getting way up there.

Out of all stable nuclides, binding energies range from 2.224 MeV for ^2H , all the way up to 1,636 MeV for ^{208}Pb . The latter may seem extremely large in comparison to the former, but binding energy should increase with more and more nucleons present, since they are providing more and more binding to the overall nucleus. The increase in binding energy with more nucleons is also seen in the values above for ^{19}F (147.8 MeV) vs. ^{53}Cr (464.3 MeV) vs. ^{75}As (652.6 MeV). Another useful parameter is the binding energy per nucleon; this is the binding energy divided by the total number of nucleons, A . This is a measure of the holding power per nucleon; the importance of this parameter is that it reflects the relative stabilities of one nuclide compared to another nuclide, even when they have different numbers of nucleons. In general, the greater the hold per nucleon, the more stable is the nucleus. To illustrate, we can calculate the binding energy per nucleon for ^{19}F .

$$\frac{147.8 \text{ MeV}}{19 \text{ nucleons}} = 7.779 \text{ MeV per nucleon}$$

Likewise calculations for ^{53}Cr and ^{75}As give 8.760 and 8.701 MeV per nucleon, respectively. Of the three nuclides, ^{75}As has the greatest total binding energy but ^{53}Cr has the greatest binding energy per nucleon. As such, ^{53}Cr would be judged to be the most stable nuclide.

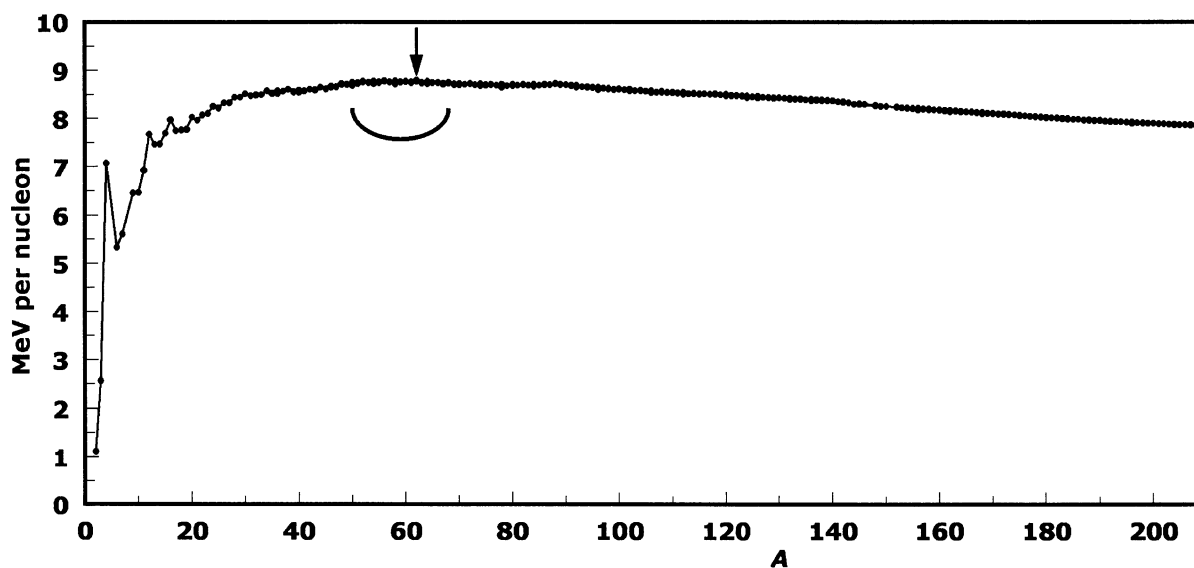
Your turn.

Example 2. Calculate the binding energy (in MeV) per nucleon for ^{84}Kr (actual mass, 83.911498 u).

You should get 8.718 MeV per nucleon.

A plot of the binding energy per nucleon vs. mass number for stable nuclides is shown below. There are no stable nuclides with $A = 5, 8, 147, 151$, and for $A = 209$ and beyond, so there are no dots for those mass numbers. The first dot is for the feeblest stable nuclide, ^2H , with 1.112 MeV per nucleon. ^3He follows ^2H , and then there is a huge jump to 7.074 for ^4He . ^4He is a special case of a very stable (doubly magic) nuclide, which is clearly out of line with its close neighbors; that stability helps to account for α emission as a common decay mode. Following a sharp drop after ^4He to ^6Li , there is then a considerable rise, albeit a bit bumpy, to the high plateau in the general region of the semioval. The absolute max is 8.795 MeV per nucleon for ^{62}Ni (at the arrow), although ^{56}Fe and ^{58}Fe are vying closely for the title with values of 8.792 and 8.790. The best nuclides are in this general region: they have the most binding energy per nucleon and they are the most stable of the stable. After these, it's slowly downhill with less binding energy per nucleon and a less stable nuclide, although there are some small bumps along the way. (Some dots in the plot may not appear circular, but those are for overlapping nuclides with the same value of A but slightly different values of MeV per nucleon.) Given these trends, and excepting some of the bumps, we can say

- for two nuclides of different mass numbers, both to the left of the high plateau, the nuclide of greater A is more stable; and,
- for two nuclides of different mass numbers, both to the right of the high plateau, the nuclide of lesser A is more stable.



There is more to this plot than what is presented here. We'll come back to this later in the Chapter after we've discussed types of nuclear reactions.

69.2 Nuclear reactions

So now we get into nuclear reactions. Keep in mind that this category of nuclear change is distinctly different from nuclear decay, as first noted in Chapter 66:

- “ Nuclear decay arises from within an unstable nuclide as a natural consequence of its own make-up. These nuclides can undergo a nuclear change on their own.

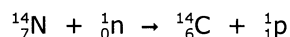
A nuclear reaction involves a nuclide reacting with some other reactant which originates from outside that atom; in other words, something else is shot into the nucleus from outside the atom. The other reactant can be another nuclide, a particle, or even a very high energy (gamma, γ) photon. ”

which was followed by:

“ Note the distinction in the above two nuclear processes: decay occurs on its own, completely within a single atom, while a reaction requires something from outside the atom. And note something else: in all prior coverage of chemical reactions, any chemical change was called a reaction. In the field of nuclear chemistry, however, a "decay" is not considered a "reaction", and these are separate categories. That distinction is a bit subtle, so keep it in mind. ”

We now begin with some new terminology and notation.

For the balanced equation for a nuclear reaction, there are typically two reactants and these are referred to as the projectile and the target. The projectile can be a γ photon or a particle such as a neutron, a proton (${}^1\text{H}$ nucleus), another small nucleus (deuterium, ${}^2\text{H}$, or tritium, ${}^3\text{H}$), α (${}^4\text{He}$ nucleus), or even some larger nucleus; that projectile is in motion towards some target. The target is some nucleus, commonly more massive than the projectile and which is often (not always) stationary. The products of the reaction typically involve a product nucleus and an emitted particle. The only equation for a nuclear reaction which we have seen thus far was in Chapter 67 for the natural production of ${}^{14}\text{C}$ in Earth's atmosphere.



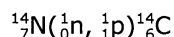
For this reaction, the ${}^{14}\text{N}$ is the target nucleus, the neutron is the projectile, the ${}^{14}\text{C}$ is the product nucleus, and the proton is the emitted particle. Note that, as for decay equations, the sum of superscripts is the same on both sides of the equation and the sum of subscripts is also the same.

Reactions such as this are often called captures. Here, the ${}^{14}\text{N}$ captures the neutron, so this is referred to as a neutron capture.

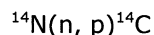
There is a shorthand notation for nuclear reactions, which takes the following form.

target(projectile, emitted particle)product nucleus

For the present reaction, this would be



but this is further shortened by removing all subscripts and removing the superscripts for the particles in parentheses.



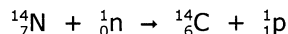
This format can be used in text, so that we can write ${}^{14}\text{N}(\text{n}, \text{p}){}^{14}\text{C}$ directly in a sentence. Additionally, the type of reaction can be indicated simply by the parenthetical part, so that we can refer to (n, p) reactions in general. There are many variations of nuclear reactions, and the following is just a sampling.

(α , γ) (α , n) (α , p) (γ , α) (γ , n) (γ , p) (n, α) (n, γ) (n, p) (p, α) (p, γ) (p, n)

Nuclear reactions and their equations can be much more complicated than decay reactions. First of all, there are four (or more!) entities in an equation for a nuclear reaction, compared to three for a decay. There are also more variations in the mass and energy picture. For radioactive decay, energy is always released, and the daughter has less mass than the parent. On the other hand, for nuclear reactions, there can be a net increase or decrease in mass and energy for the products relative to the reactants; the difference can be accommodated by the kinetic energies (velocities) of the projectile and the emitted particle, or by the energy of the photon if γ is involved. There can also be very large energy barriers to overcome. For example, α particles carry a 2+ charge, and hitting a nucleus with a positive (often, a very positive) charge requires very high kinetic energies for the α particles in order to overcome strong charge repulsion. For a proton projectile, there is less charge repulsion but it's still substantial. For neutrons, there is no charge repulsion, and neutrons can react at a wide range of kinetic energies. For γ , there is again no charge repulsion; as a photon, its speed is fixed, and differences in frequency provide differences in energy. As can be seen, the kinetic energy of the projectile (and even of the emitted particle) can play a very important part in the energy picture, and projectiles must often hit a target at extremely high speeds for a particular reaction to occur. Another complication which can be mentioned is the

involvement of an intermediate. In general, the impact of the projectile with the target nucleus can result immediately in the ejection of one or more particles or photons; alternatively, that first collision product can last a fraction of a second as an intermediate, while the energy of impact distributes throughout all nucleons in the nucleus. In this latter case, that intermediate is called a compound nucleus, which then undergoes some process to form the final product(s) of the reaction. It is also noted that the products of nuclear reactions are often radioactive; they eventually undergo their own decay, but that decay is separate from the nuclear reaction which formed them.

To make matters more manageable among the numerous nuclear processes possible, we will avoid some of the details of kinetic energies, compound nuclei, and other complications which can arise, although there will yet be some mention of these in select cases below. We will work primarily with the identities of the particles and the nuclides which are involved. We can work with the equations for nuclear reactions as illustrated above in the two formats as shown. We started with

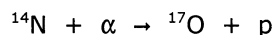


and this format will be called full format. The shorthand version, as given by ${}^{14}\text{N}(\text{n}, \text{p}){}^{14}\text{C}$, will be called short format.

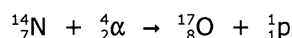
Let's bring in another reaction for illustration. While Nature has been doing nuclear reactions throughout the universe for billions of years, the first nuclear reaction performed by humans was reported in the early 20th century and involved ${}^{14}\text{N}(\alpha, \text{p}){}^{17}\text{O}$. Let's write out the full equation for this reaction. In order to do so, note that

${}^{14}\text{N}$ is the target and α is the projectile, and these will appear on the left side of the equation, in either order; and,
 p is the emitted particle and ${}^{17}\text{O}$ is the product nucleus, and these will appear on the right side, in either order.

Set these up



and then bring in the missing superscripts and subscripts.

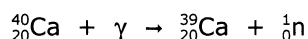


That's it.

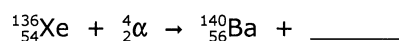
Converting between full and short formats is fairly straightforward. We can also leave one nuclide or one particle or a γ out of the given information, and you need to identify that. Let's take a gander at a few of these variations.

Example 3.

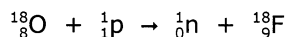
- A. One way to prepare ${}^{18}\text{F}$ for PET scans is via ${}^{18}\text{O}(\text{p}, \text{n}){}^{18}\text{F}$. Write the full equation for this reaction.
 B. Write the short format equation for the following reaction.



- C. Fill in the blank for _____(p, α) ${}^{53}\text{Mn}$ and write the full equation for this reaction.
 D. For the following reaction, fill in the blank and write the short format equation for the reaction.



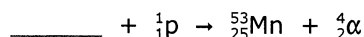
A. OK, the ${}^{18}\text{O}$ and p go on the left side of the equation, and n and ${}^{18}\text{F}$ go on the right. Set that up, and put in subs and supers where they're missing.



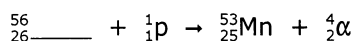
Done. As a check, note that the superscripts sum to 19 on each side, while the subscripts sum to 9.

B. The target is ${}^{40}\text{Ca}$, the projectile is γ , the emitted particle is n , and the product nucleus is ${}^{39}\text{Ca}$. This comes to ${}^{40}\text{Ca}(\gamma, \text{n}){}^{39}\text{Ca}$.

C. In order to identify the missing target, you need to go through supers and subs. Here's what you have so far.



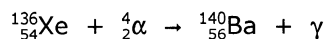
The sum of superscripts on the right is 57; this sum must also be 57 on the left. The sum of subscripts on the right is 27; this sum must also be 27 on the left. Enter the necessary values for the unknown.



Who is #26?



D. The superscript of the unknown would be zero and the subscript of the unknown would be zero, but that means no mass number and no charge. That can only be a photon, namely γ .



For the short notation, ${}^{136}\text{Xe}$ is the target, α is the projectile, γ is the emitted particle, and ${}^{140}\text{Ba}$ is the product nucleus. That comes to ${}^{136}\text{Xe}(\alpha, \gamma){}^{140}\text{Ba}$.

A wide variety of nuclear reactions find practical use in the modern world. If you need a radioactive isotope to do some job, then most of those have to be made. These preparations often involve nuclear reaction steps, and some also involve radioactive decay steps somewhere along the way.

There remain more types of nuclear reactions than the ones mentioned so far. Two other categories, very important to human society and/or to the universe as a whole, are fission and fusion. Each of these gets its own discussion.

69.3 Fission

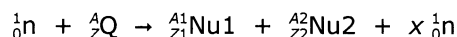
We first got into fission in Chapter 66.

“ In general, fission is the splitting of one larger nucleus into a pair of two smaller nuclei, commonly accompanied by emission of neutrons and γ . There are two types, spontaneous fission and induced fission. Spontaneous fission happens to a nucleus on its own; this is a nuclear decay process. Induced fission is triggered to happen by something shot into the nucleus from outside the atom; this is a nuclear reaction. We will come back to induced fission later in Chapter 69, where additional aspects of fission will be presented. ”

Later is now.

As a nuclear reaction, induced fission is more complicated than the other nuclear reactions just discussed above. For one thing, there are now two (or, rarely, three) product nuclides and there can be up to a few emitted particles. There can also be multiple gamma emissions, although those will not be our focus here. The most common projectile is a neutron; again, a neutron has no charge repulsion to overcome. As such, fissions can be initiated using a wide range of neutron kinetic energies, depending on the intended reaction.

As noted in Chapter 66, fissions are inherently messy, and the outcome of a single event is unpredictable. A general reaction for neutron-induced fission can be depicted as



wherein Q is the target nucleus, Nu1 and Nu2 are the product nuclei, and x is typically 0 - 5 (but can be more). This equation is similar to the equation given for spontaneous fission in Chapter 66 but now it includes a projectile neutron on the left. Note that neutrons appear on both sides of the equation. For the many balanced equations in all prior Chapters for chemical reactions, we had always cancelled like items on both sides of the equation. For fission equations, however, they are retained on both sides to convey the manner of conducting the reaction. Thus, for the equation as shown, we know one neutron is the projectile and the reaction results in some number, x, of neutrons being freshly emitted.

As in Chapter 66, the element identities of the product nuclei, Nu1 and Nu2, must comply with the following.

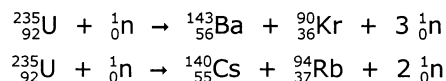
$$Z = Z_1 + Z_2$$

The specific isotopes of those elements, as well as the number of ejected neutrons, must satisfy

$$1 + A = A_1 + A_2 + x$$

which now has a 1 on the left side, relative to the Chapter 66 version, due to the projectile neutron. All of this again leads to many options for A1, A2, x, Z1 and Z2. Not all arithmetic combinations are allowed

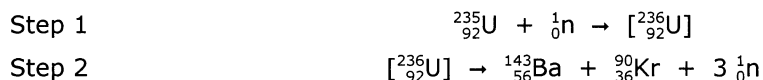
due to some constraints, but it's still a large number of different options. For example, consider two possible fission reactions for a neutron with ^{235}U .



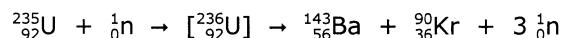
That's just two. There are hundreds of other possible product combinations.

Let's go into more detail on fission, using ^{235}U for illustration purposes due to the importance of these reactions to modern society in terms of nuclear energy and, unfortunately, of weaponry. A key feature for ^{235}U is that it reacts readily with low energy neutrons, whereas other targets can require neutrons of much higher energy.

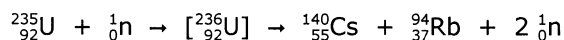
I will bring in the role of a compound nucleus as an intermediate for this illustration. We can depict the first reaction above in two Steps as



where the brackets indicate that the compound nucleus is generated in some high energy state. It could undergo γ emission to give ground state ^{236}U , which corresponds to an overall $^{235}\text{U}(n, \gamma)^{236}\text{U}$ reaction. Or, the $[{}^{236}\text{U}]$ compound nucleus can undergo fission. For writing purposes, we can combine the two fission Steps above into one line as in following,



and likewise for the other reaction mentioned above involving a different set of products.



The equations show 3 and 2 neutrons are produced for every one neutron used up. (This will be important below.) These are the most common numbers for neutron-induced fission of ^{235}U . Over huge numbers of fission events involving hundreds of product combinations, the average number of neutrons released per single fission event is 2.47.

Fissions are inherently messy. The $[{}^{236}\text{U}]$ is in a high energy and seriously unstable condition. This allows a sizable chunk of nucleons to pull away from the rest of the nucleus. As the two parts further separate, the connection stretches and narrows, and it finally pinches off to give two "fission fragments". At the moment of disconnect, there is no longer any strong force holding the two fragments together. (Keep in mind that the strong force acts at close contact between immediate neighbors, but charge repulsion is felt over greater distances.) At the point of disconnect, the full force of charge repulsion kicks in, with both of the new fragments repelling each other in an extremely energetic way. The separated, yet-frenzied fragments can then emit neutrons, as shown in the above equations. (Photons can also be emitted.) The resulting fission products typically display an asymmetric (uneven) split in their mass numbers; this is seen in the two equations above, wherein ^{143}Ba and ^{140}Cs are considered the heavy products and ^{90}Kr and ^{94}Rb are considered the light products for their respective reactions. Common mass ranges for the split of $[{}^{236}\text{U}]$ are $A = 131 - 150$ for the heavy products and $A = 86 - 106$ for the light products. There are numerous combinations outside those ranges but they are in small amounts. Symmetric (even) splits can happen, giving two product nuclides around $A \sim 118$; for $[{}^{236}\text{U}]$, however, symmetric splits are rare. On the other hand, symmetric splits are more common in higher energy fissions or in fissions starting from other target nuclides.

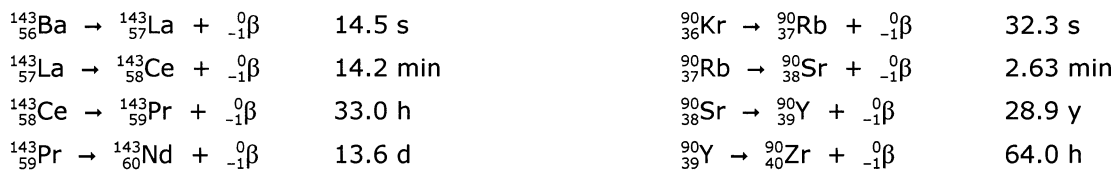
And it ain't over yet!

Most product nuclides are themselves radioactive, so decays can follow. Actually, decay chains can follow. From the stability discussion in Section 66.4, recall that stable nuclides must have $Z < 83$ and must have some N/Z ratio which is neither too high nor too low. ^{236}U (which has too many protons to be stable) starts off with $N/Z = 1.57$. Splitting that will liberate several neutrons immediately, but the resulting fission fragments will still have an N/Z ratio around 1.5. That is way too high for the much lighter fragments. As such, they need to lose neutrons and/or gain protons, and β^- decay does just that. That situation was addressed near the end of Section 66.4,

“ For a radionuclide above the stable dots, the N/Z ratio is too large. Many of these decay by β^- emission, which decreases N by one and increases Z by one; thus, the N/Z ratio decreases.”

For each fission fragment, a series of β^- decays typically takes place until a stable nuclide is obtained. Each step increases Z by one, decreases N by one, and lowers N/Z (while keeping the same A). To

illustrate, here are the decay chains for the $^{143}\text{Ba} + ^{90}\text{Kr}$ pair cited above; these chains end at ^{143}Nd and ^{90}Zr , which are stable. The half-lives are also listed.



Here is the breakdown for how the N/Z ratio decreases for each step of decay.

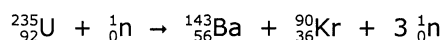
	$^{143}_{56}\text{Ba}$	\rightarrow	$^{143}_{57}\text{La}$	\rightarrow	$^{143}_{58}\text{Ce}$	\rightarrow	$^{143}_{59}\text{Pr}$	\rightarrow	$^{143}_{60}\text{Nd}$
$N:$	87		86		85		84		83
$Z:$	56		57		58		59		60
$N/Z:$	1.55		1.51		1.47		1.42		1.38

	$^{90}_{36}\text{Kr}$	\rightarrow	$^{90}_{37}\text{Rb}$	\rightarrow	$^{90}_{38}\text{Sr}$	\rightarrow	$^{90}_{39}\text{Y}$	\rightarrow	$^{90}_{40}\text{Zr}$
$N:$	54		53		52		51		50
$Z:$	36		37		38		39		40
$N/Z:$	1.50		1.43		1.37		1.31		1.25

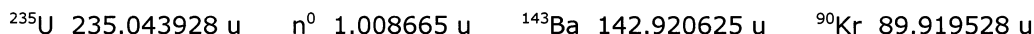
As can be seen, there can be a lot of activity following the original fission.

Did I mention fissions are inherently messy?

Fissions can involve very large amounts of energy. We can calculate this for the first fission equation from above



and we'll solve for MeV. We need some masses.



Here's the layout. We can drop the mass of one neutron from both sides to simplify.

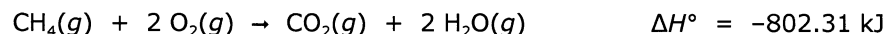
$$\Delta m = 142.920625 \text{ u} + 89.919528 \text{ u} + (2 \times 1.008665 \text{ u}) - 235.043928 \text{ u} = -0.186445 \text{ u}$$

$$E = 0.186445 \text{ u} \times \frac{931.5 \text{ MeV}}{\text{u}} = 173.7 \text{ MeV}$$

That's huge. Let's bring that up to one mole scale via $E = mc^2$.

$$E = mc^2 = (1.86445 \times 10^{-4} \text{ kg}) \times (3.00 \times 10^8 \text{ m/s})^2 = 1.68 \times 10^{13} \text{ J} = 16.8 \text{ TJ}$$

This is the energy released for just one of hundreds of different fission reactions. Furthermore, the subsequent chains of β^- decays will release even more energy. A typical value of ~ 200 MeV is commonly cited for the total energy released from the induced fission of one atom of ^{235}U plus the subsequent β^- decays; this comes out to ~ 19.3 TJ on the one mol scale. This is a massive amount of energy from the overall fission process, and this accounts for the use of fission in nuclear energy production. Let's compare that to fossil fuel combustion as an energy source using CH_4 , which is the principal component in natural gas.



A release of 802.31 kJ by combustion is abysmally minuscule in comparison. These numbers are based on moles of reactant(s) for the reaction as written; let's compare the two based on one gram of total reactant(s): for U fission, this means one gram of U; for CH_4 combustion, this means one gram of a stoichiometric mix of $\text{CH}_4 + 2 \text{O}_2$. (For purposes of this calculation, the stoichiometric mix of $\text{CH}_4 + 2 \text{O}_2$ has a total mass of $(16.04 \text{ g} + 2 \times 32.00 \text{ g} = 80.04 \text{ g})$, and we are taking one gram of that.)

One gram of ^{235}U undergoing induced fission and subsequent decays would yield ~ 82.1 GJ.

One gram of a stoichiometric mix of $\text{CH}_4 + 2 \text{O}_2$ undergoing combustion would yield 10.02 kJ.

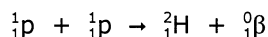
The ratio is 8.19 million: it would take the combustion of $8.19 \times 10^6 \text{ g}$ (9.03 tons) of the $\text{CH}_4 + 2 \text{O}_2$ mix to meet the energy equivalent of the fission of one gram of ^{235}U . This is a comparison of nuclear energy versus chemical energy. There is far greater potential for nuclear energy relative to fossil fuel energy for powering human societies, and ^{235}U is the leading fuel used in nuclear reactors. Despite the greater positive potential, there are also substantial negatives. Operations of nuclear plants have incurred

accidents which can impact the health of thousands, along with large areas of land, sea and air. Furthermore, there is a formidable waste problem: nuclear reactors generate tons of radioactive waste, which can include structural components of the reactor as well as spent fuel components. The waste contains vast numbers of different radioactive nuclides, due to the messiness of the overall fission process. This waste amounts to a total of hundreds of thousands of tons globally over the years. This must be safely stored to allow for sufficient decay time, which can be decades or more for some nuclides, and then up to 100,000 years for others. There are reprocessing/recycling efforts available for some waste, and some of these products even make their way into nuclear medicine and other beneficial nuclear products. On the other hand, there is far greater tonnage of waste from fossil fuel combustion in the form of CO_2 , which is simply spewed into the atmosphere and which can dissolve into oceans and other waterways worldwide, wreaking havoc to global climate and ecology. Coal is far worse than natural gas due to other components present and the amassing of coal ash waste. As can be seen, there are very complicated issues involved in energy production. Alternative energy sources have been increasing in use, but these yet pale in comparison to fossil fuel and nuclear energy production on a global basis.

Let's return to the point I made upstairs regarding the neutron count. On average, 2.47 neutrons are released for every fission event which started with one neutron. Thus, on average, more neutrons are produced than are used up, and these emitted neutrons can then initiate additional fissions with other ^{235}U nuclei. This sets up a chain of fission events, referred to as a chain reaction. This could greatly accelerate the rate of fissions occurring and, under certain conditions, that gets into a bomb scenario. But some of those excess neutrons can be captured by "control rods" so that the rate of fission can be controlled; this is the nuclear reactor scenario. Commonly, ^{113}Cd or ^{10}B are used in such control rods because these nuclides have a very high rate of neutron capture, leading to $^{113}\text{Cd}(n, \gamma)^{114}\text{Cd}$ and $^{10}\text{B}(n, \alpha)^7\text{Li}$ reactions.

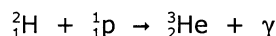
69.4 Fusion

Fusion is the joining of two smaller nuclei to make one larger nucleus, often accompanied by a release of particles and/or photons. We can consider the simplest of these to be the reaction of two protons (^1H nuclei) to produce a deuterium nucleus and a positron.

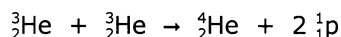


This reaction releases 0.419 MeV. Unlike induced fission with neutrons, fusion reactions do have an energy barrier as a result of charge repulsion, since two positively charged nuclei must collide. For this reaction, it is possible to get around the charge barrier at very high temperatures, but the barrier can be more extreme in fusions involving nuclei with higher positive charges.

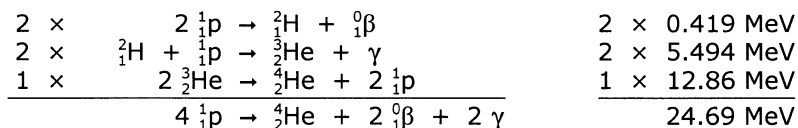
This simple reaction is the starting point for the energy of the Sun and similar stars. The interior of the Sun has a temperature of $\sim 1.6 \times 10^7$ K, at which T all electrons are stripped from atoms to give a plasma composed of naked nuclei, individual protons, individual electrons, and other particles. There are several fusion reactions occurring; for example, a deuterium from the above reaction can fuse with another proton to produce ^3He



and this releases 5.494 MeV. Two ^3He nuclei can then fuse to form ^4He and return a couple of protons



which releases 12.86 MeV. These are the most important reactions in the Sun. The three equations can be summed in the following manner



and this provides an overall equation for the net synthesis of ^4He from four protons. This process is referred to as hydrogen "burning", and this sequence accounts for 91% of the Sun's energy. 24.69 MeV is a helluva lot of energy for these lightweights. Let's return to our prior comparison of fission and combustion energies, and compare all three on the basis of one gram of total reactants.

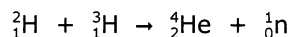
One gram of ^1H undergoing fusion to ^4He would yield 592 GJ.

One gram of ^{235}U undergoing induced fission and subsequent decays would yield ~ 82.1 GJ.

One gram of a stoichiometric mix of $\text{CH}_4 + 2\ \text{O}_2$ undergoing combustion would yield 10.02 kJ.

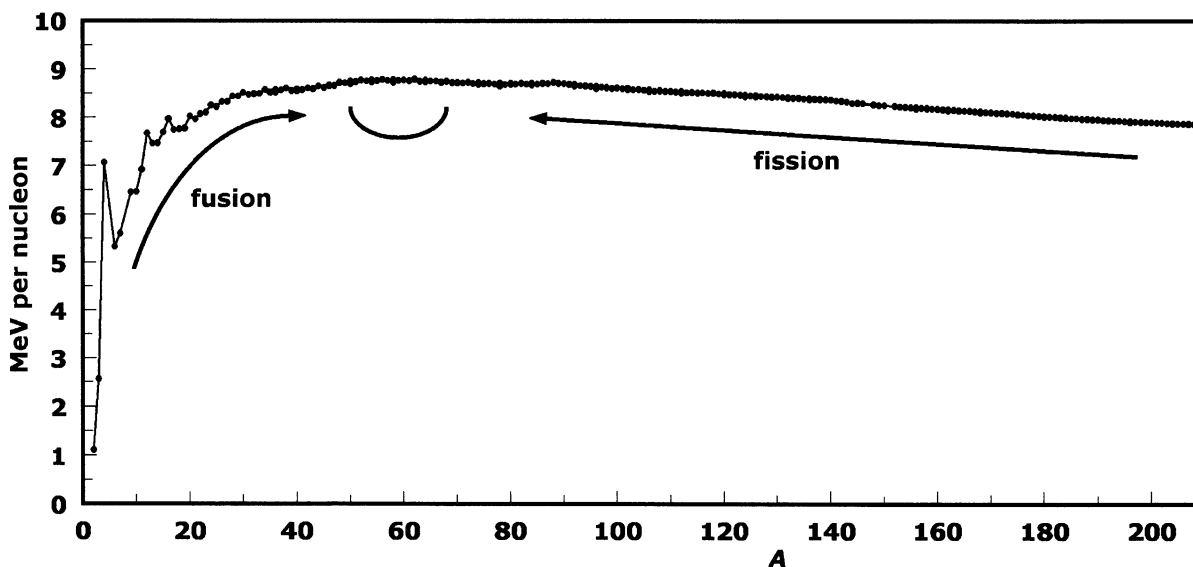
The magnitude of fusion energies on a mass basis are absolutely stellar.

With the huge amount of energy available from fusion processes, there is tremendous interest in tapping into that to serve human society. Unfortunately, controlled fusion is not yet a usable energy form (although uncontrolled fusion provides weaponry). The lead candidate reaction which has been most investigated is the reaction of deuterium and tritium



which needs a temperature in the range of $\sim 10^8$ K and which releases 17.59 MeV (which corresponds to 338 GJ for one gram of a stoichiometric mix of ${}^2\text{H} + {}^3\text{H}$). There are serious technological hurdles to initiating and sustaining the reaction, however, due to the extreme temperatures necessary. Several system designs have been under study for decades. A notable feature of a fusion reactor relative to a fission reactor is that there is no waste from the fuel to store for thousands of years, although structural components can become radioactive due to neutron bombardment. Nevertheless, the radioactive waste products from a fusion reactor have much shorter half-lives than those from a fission reactor, and storage times of decades instead of 100,000 years may be adequate.

We will see much more of fusion in the upcoming Section but, right now, I want to tie fission and fusion back to the notion of binding energies per nucleon, as discussed earlier in this Chapter. Again, a higher binding energy per nucleon is an indicator of better stability, and any process which results in more binding energy per nucleon will release energy. Below is a revision to the plot which had been given previously, now showing the general regions in which fusion and fission are energy-releasing. To the left



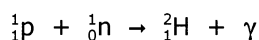
of the high plateau of stability, the fusion of two nuclei leads to a nucleus of greater A which typically gives more MeV per nucleon, and therefore greater stability. As such, fusion in this range releases energy. To the right of the plateau, fission of a nucleus produces two nuclei of lower A ; this typically results in more MeV per nucleon for each fragment, and this is also energy-releasing. Thus, approaching the high plateau from either direction releases energy.

Let's get back to Old Sol. And to some distant relatives.

69.5 Nucleosynthesis, or a very Big Bang and beyond

We've been talking about the many chemical elements and their isotopes since Chapter 2 but we haven't really said much about where they came from. So where did they come from? That's the subject of nucleosynthesis. Nucleosynthesis involves the production of elements from different elements. Fusion is a big part of this process, and so also is radioactive decay.

The universe began an estimated 14 billion years ago from a Big Bang which unleashed an unimaginable fury of energy, which gave rise to subatomic particles at untold temperatures ($> 10^{30}$ K), far in excess of anything at which normal matter could exist. Expansion and cooling followed immediately and rapidly. At $\sim 10^{11}$ K, protons, neutrons, electrons and other subatomic particles existed, as well as photons, but the temperature was too great for even simple nuclei to form. That would take several minutes, when the temperature had cooled to $\sim 10^9$ K. Deuterium was the first new nucleus to form



and subsequent reactions of ${}^2\text{H}$ with other protons and neutrons led to ${}^3\text{He}$ and ${}^3\text{H}$, respectively. Those nuclei likewise reacted in various fashions to produce ${}^4\text{He}$, while direct fusion of two ${}^2\text{H}$ also produced ${}^4\text{He}$. Some ${}^7\text{Li}$ was also produced along the way, but it was prone to decomposition by (p, α) or (γ, α) reactions. After the universe further expanded and cooled over 30 min, fusion ceased and nucleosynthesis was at a standstill. For millions of following years, the composition of the elements in the universe was

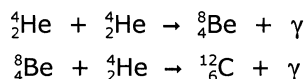
~76% hydrogen, mostly ${}^1\text{H}$ with traces of ${}^2\text{H}$, while ${}^3\text{H}$ disappeared due to its own radioactive decay ($t_{1/2} = 12.3 \text{ y}$);
 ~24% helium, mostly ${}^4\text{He}$ with traces of ${}^3\text{He}$; and,
 scant traces of ${}^7\text{Li}$.

Free neutrons had disappeared as they underwent their own decay ($t_{1/2} = 10.2 \text{ min}$). At first, the elements were all bare nuclei. Free electrons were available, but temperatures were too high for electrons to bind to nuclei until after the first million years or so.

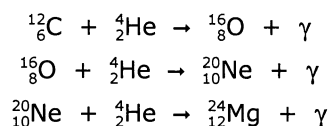
Thus, the Big Bang provided H, He and traces of Li, but that was about it. Eventually, over millions of years afterwards, gravity would gather the widely dispersed matter into galaxies and then into the first generation of stars within those galaxies. While expansion had provided cooling, gravitational compaction provided heat to a forming star, which ultimately got hot enough for fusion to start in its core, thereby releasing more heat. Over the ensuing billions of years, stars were the hotbed of nucleosynthesis: by far, most heavier elements have arisen from the lives and deaths of stars, a process which continues today.

Let's take a look at a few of the processes within stars which have been relevant to nucleosynthesis over the eons. The star's mass is a major factor which determines its lifetime, what temperatures it can reach, what elements it can form, and whether it ultimately explodes (supernova) or, less impressively, slowly loses its layers. More massive stars burn brighter and burn out faster. Our Sun's mass is $2.0 \times 10^{30} \text{ kg}$ and its core density is $\sim 1.5 \times 10^5 \text{ kg/m}^3$; as noted earlier, its core temperature is $\sim 1.6 \times 10^7 \text{ K}$. A long life yet awaits it. Core density and temperature are very important to a star. Higher T provides for higher kinetic energies of particles and nuclei, thus allowing heavier nuclei to form. Higher T also acts to expand the core but that is opposed by the gravitational attraction which acts to compact the core.

The hydrogen burning sequence given for the Sun in the prior Section is a very typical lead-in to the start of stellar nucleosynthesis. After the hydrogen is consumed in a star, gravitational collapse occurs, increasing the core temperature to $\sim 10^8 \text{ K}$. (Our Sun is not there yet.) At this time, the star is a red giant; helium can burn, which leads to heavier elements. Carbon forms in a two-step sequence.

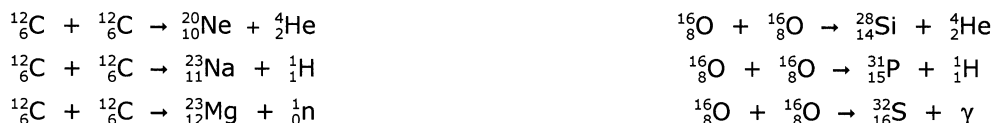


${}^8\text{Be}$ can very rapidly decay back to two α , but its fleeting existence and the massive abundance of ${}^4\text{He}$, along with some favorable energetics, yet allow for the synthesis of ${}^{12}\text{C}$ by this route. More α -capture reactions can follow



but note that these involve collisions of ${}^4\text{He}$ with increasingly positive nuclear charges. Thus, each step faces a greater charge barrier for fusion, requiring higher and higher temperatures.

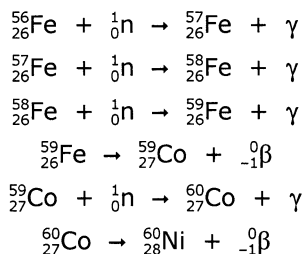
Eventually, the He runs out. Gravitational collapse again occurs, accompanied by further increases in temperature. In the $\sim 10^9 \text{ K}$ range, ${}^{12}\text{C}$ and ${}^{16}\text{O}$ can start their own fusions, but now the products are more variable. The following are examples.



With increasingly higher temperatures, other processes can also occur. Overall, fusion forges elements up to the high plateau of MeV per nucleon, the pinnacle of nuclear stability. Fusions of positively charged nuclei up to that range release energy, but fusions beyond that range are not energetically favorable.

To go further, other mechanisms come into play.

A common mechanism for nuclear masses beyond the high plateau involves a series of (n, γ) reactions and β^- decays. Again, neutrons have no charge barrier to overcome while flying into some nucleus. On the other hand, the (n, γ) reactions are limited by the availability of neutrons. There are no large abundances of neutrons available in a typical star, because free neutrons are unstable. But neutrons are constantly produced in some of the other ongoing reactions, and this allows neutron capture to still occur, although it is slow. This sequence is called the s-process, wherein the 's' is for slow. As an example, here are a few steps of a sequence starting from the fusion product ^{56}Fe and ending at ^{60}Ni .



These sequences can go on and on, involving over 100 steps. Most naturally occurring elements from $A = 56 - 210$ can be prepared in this manner. A key feature here is that, since neutrons are in short supply, many of the radioactive nuclei which are capable of beta decay will do so before having a chance to pick up another neutron.

An alternative to the slow s-process is the rapid r-process. This is somewhat similar to the s-process but now there is a huge abundance of neutrons. These are not typical conditions, but such conditions can occur during rare mergers of neutron stars or in supernova explosions. In contrast to the s-process, neutron capture in the r-process is very fast, with many neutron captures occurring in seconds. Under these conditions, neutron capture is now much faster than the radioactive decay. This paves the way for yet more nuclides, even those with $A > 250$. All elements beyond Pu, however, have half-lives much less than the millions or billions of years since they were synthesized; as such, those elements decayed before they could accumulate to a significant extent. Nevertheless, their synthesis steadfastly continues.

The reactions described here are some of the steps of stellar nucleosynthesis, and there are plenty of others. But it is not until the star slowly withers or dies explosively that the elements made within that star are released to the Universe, where they might eventually be gathered up again as part of a newer generation of star or even as a planet. These are unmistakably slow processes: after ~ 14 billion years, a mere 2% (by mass) of the original H + He has been converted to all other elements combined.

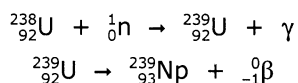
On a much smaller scale, humans have also gotten into the act, forging nuclides which are not naturally found on Earth.

69.9 Transuranium elements and the superheavies

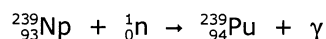
Humans have been making new isotopes of known elements, and even making new elements, since the 1940s. The focus of the present discussion is for the largest of these, known as transuranium elements and superheavy elements. The transuranium elements are all elements with atomic number greater than 92 (U), and which include the superheavy elements. The superheavy elements are commonly, but not always, defined as starting with rutherfordium, Rf, ($Z = 104$) and extending to all higher Z . All of the elements in these categories are radioactive, with half-lives ranging from microseconds to millions of years. None of these have a significant natural abundance on Earth. A number of the transuranium elements, such as ^{241}Am , have a very important role in human society.

Let's look at several approaches for the syntheses of the transuranium elements.

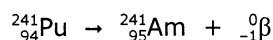
An early successful pathway involved combinations of neutron captures and β^- decays. For example, starting from naturally abundant ^{238}U , one can prepare neptunium, ^{239}Np .



^{239}Np can be used to prepare plutonium, ^{239}Pu ,

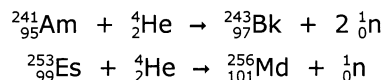


which finds use as the second most important fuel in nuclear power reactors. Two more (n, γ) steps get you ^{241}Pu which decays to americium, ^{241}Am ,

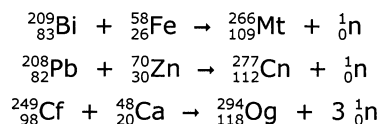


which then shows up in your smoke detector. Such series can continue for over a dozen steps, producing various isotopes of these elements, as well as isotopes of curium (Cm), berkelium (Bk), californium (Cf), einsteinium (Es) and fermium (Fm).

A second pathway involves bombardment by light, charged particles, primarily ${}^4_2\text{He}$. The original syntheses of Cm, Bk, Cf and Md (mendelevium) used this route. Here are two examples.



In general, as nucleosynthesis continues into heavier elements, especially well into the superheavies, more drastic measures become necessary. These require more highly charged particles slamming into highly charged targets, at extreme energies, in more and more powerful particle accelerators, in order to overcome more extreme charge barriers. Many of these operations may actually have very little success, with some elements being thus prepared at a rate of a few atoms per day or even a few months per atom. A primary problem in many cases is that a compound nucleus forms in a very excited state and it must dissipate a lot of energy quickly in order to survive or it is likely to self-destruct by fission. It can relax by emitting neutrons and/or photons and this maintains the new element's identity. On the other hand, fission would change the new element's identity. If the atom does survive its own formation, it will decay, most commonly by α decay or spontaneous fission, followed in many cases by a chain of further decay steps. Here are examples of the first reported syntheses for meitnerium (Mt), copernicium (Cn) and oganesson (Og).



The half-lives of several of the superheavies run down to μs range; the element may never be observed directly, but it is identified by backtracking through the observed decay chain.

Whither now?

Oganesson filled out the Seventh Period of the Periodic Table. Efforts to prepare new superheavy elements continue but experimental methodologies become more limited and further progress has been slow. It is believed that targeting magic numbers for protons and neutrons might lead to new elements. Unfortunately, superheavy nuclei tend to be deformed, and there is uncertainty in the values of magic numbers for deformed nuclei. Besides the search for new elements, there is interest in developing better methods of synthesizing the ones that are known, as well as synthesizing isotopes of those elements with longer, and more practical, half-lives.

There is tremendous interest in the chemistry of the newer elements, although this is very difficult to study when atoms are produced which only last for fractions of a second. But progress has been made with some of the longer-lived elements, and numerous chemical compounds have been prepared. Computational methods can provide some information on the chemistry of these elements, no matter how fleeting their existence. Some of the chemistry is expected to be very different for some of the superheavy elements compared to their Group members higher in the Periodic Table. For example, Og may be more reactive than Cn, in stark contrast to the general trend of much greater reactivity for Group 12 than for the noble gases. Also, and very importantly (and as briefly noted in Section 22.2), electrons in atoms with $Z > 120$ will begin to populate an electronic g subshell, namely $5g$, and that could also lead to different chemistry.

There are plenty of new frontiers waiting to be explored! Muß es sein? Es muß sein.

Problems

The following values for masses (g or u) are needed.

${}^1_0\text{n}$ 1.008665	${}^4_2\text{He}$ 4.002603	${}^{65}_{29}\text{Cu}$ 64.927790	${}^{140}_{56}\text{Ba}$ 139.910608
${}^1_1\text{H}$ 1.007825	${}^{11}_5\text{B}$ 11.009305	${}^{96}_{38}\text{Sr}$ 95.921719	${}^{239}_{94}\text{Pu}$ 239.052162

- True or false.
 - In general, a greater binding energy per nucleon represents a greater stability.
 - Of the stable nuclides ^{24}Mg and ^{39}K , ^{24}Mg has a greater binding energy per nucleon.
 - Of the stable nuclides ^{108}Pd and ^{127}I , ^{127}I is more stable.
 - A (γ, p) reaction for some target element produces an isotope of that element.
 - An (α, p) reaction for some target gives a product nucleus with one more proton than the target.
 - Following the neutron-induced fission of ^{235}U , the fission fragments typically undergo β^- decays until a stable nuclide is reached.
 - Most of the Sun's energy is due to hydrogen burning.
- For each of the following, calculate the mass defect (in u), the binding energy (in MeV) and the binding energy (in MeV) per nucleon.
 - ^{11}B
 - ^{65}Cu
- For each process below, identify the target or product nucleus as given by the blank, and write the full equation for each.
 - _____ $(\alpha, \gamma)^{242}\text{Pu}$
 - $^{40}\text{Ca}(\gamma, p)$ _____
- For each reaction below, identify the target or product nucleus as given by the blank, and write the short format for each.
 - $^4_2\alpha + \text{_____} \rightarrow ^{13}_6\text{C} + ^1_1\text{p}$
 - $^{197}_{79}\text{Au} + \gamma \rightarrow ^1_0\text{n} + \text{_____}$
- The reactions and decay below are used to prepare a radionuclide (indicated by the blank space) which is used in medical applications. (The type of medical application is indicated in parentheses.) Identify each radionuclide.
 - $^{44}\text{Ca}(p, n)$ _____ (PET scans)
 - $^{32}\text{S}(n, p)$ _____ (cancer treatment)
 - $^{176}\text{Yb}(n, \gamma)$, followed by a β^- decay to give _____ (cancer treatment)
- The following is a possible fission reaction for neutron-induced fission of ^{239}Pu . Calculate the energy for this process in MeV and calculate the energy in TJ for one mol.

$$^{239}_{94}\text{Pu} + ^1_0\text{n} \rightarrow ^{140}_{56}\text{Ba} + ^{96}_{38}\text{Sr} + 4 ^1_0\text{n}$$
- Consider part of an s-process sequence of nucleosynthesis starting from ^{104}Pd . The sequence involves five neutron captures, one β^- decay, one more neutron capture and then one more β^- decay. At this point, what nuclide has been formed?